Critical exponents of self-avoiding walks on a family of truncated *n*-simplex lattices

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ADDENDUM

Critical exponents of self-avoiding walks on a family of truncated n-simplex lattices

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Abstract. The values of critical exponents of self-avoiding walks on an n-simplex lattice with $n \to \infty$ are evaluated.

Recently we developed an approximate theory for critical exponents of self-avoiding walks (SAWs) on n-simplex lattices when n is large [1]. The purpose of this communication is to calculate the values of the exponents in the limit $n \to \infty$. The notation adopted here is that of [1].

The recursion relation for the swollen state in the limit of large n, is [1]

$$A_{r+1} = A^2 + (n-2)A^3 + (n-2)(n-3)A^4 + \ldots + (n-2)!A^n. \tag{1}$$

As usual, the subscript r, which indicate the level of iteration, is dropped. Differentiation of (1) at its fixed point, with respect to A^* gives the eigenvalue

$$(n+1) - (1/A^*) \tag{2}$$

where A^* is the fixed point of (1). The series of (1) is summed to give

$$A_{r+1} = (n-2)!A^n \left[\exp(1/A) - \sum_{l=n-1}^{\infty} (1/l!A^l) \right]. \tag{3}$$

It is easily seen that the second term in the square bracket of (3) is negligible compared to the first in the limit $n \to \infty$. The term with maximum contribution in the series of (1) is found to be $m \simeq (1/A)$ and the width of the maximum $\sqrt{m}$. Since in the limit $n \to \infty$,

$$(1/A) + (1/A^{1/2}) \ll n - 1$$

the contribution arising from the second term in (3) is limited to the region far beyond from the maximum and goes to zero as $1/n^2$. The fixed point of (3) is found to be

$$A^* = (1+s)/(n+1) \tag{4}$$

for $n \to \infty$, where

$$s \sim ((\ln n)/n)^{1/2}. \tag{5}$$

Substituting (4) and (5) into (2), we find

$$\lambda = (n+1)^p \tag{6}$$

with

$$p = \tfrac{1}{2}[1 + (\ln(\ln n)/\ln n)]. \tag{7}$$

Note that the value of p given in [1] was determined empirically by fitting the data reported in table 1 of [1]. Equation (7) gives the value of p which is good in the limit $n \to \infty$ and may not reproduce the eigenvalues given in the table 1 of [1].

The relation for exponents ν, α and γ given in terms of p by (5.10) and (5.12) of our previous work [1], however, remain valid in the limit $n \to \infty$. When we substitute the value of p given by (7) into these equations we find

$$\nu \sim (\tilde{d}/\bar{d})[1-(\ln(\ln n)/\ln n)] \tag{8}$$

$$\gamma \sim \tilde{d}[1-(\ln(\ln n)/\ln n)]. \tag{9}$$

Thus in the limit $n \to \infty$ we find $\nu \sim (2/\bar{d})$ and $\gamma \to 2$. The scaling relation,

$$\alpha + \gamma = 2 \tag{10}$$

remains unchanged.

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Reference

[1] Kumar S, Singh Y and Joshi Y P 1990 *J. Phys. A: Math. Gen.* **23** 2987